[2]
$$\boxed{\begin{aligned}(2D-2)[x]+(D-2)[y]&=-e^{-2t} \quad ①\\ (5D+4)[x]+(3D+2)[y]&=2e^{-2t} \quad ②\end{aligned}}$$
$2\frac{1}{2}$

$(3D+2)[①]:\ (3D+2)(2D-2)[x]+(3D+2)(D-2)[y]=(3D+2)[-e^{-2t}]$

$-(D-2)[②]:\ -(D-2)(5D+4)[x]-(D-2)(3D+2)[y]=-(D-2)[2e^{-2t}]$

$$((6D^2-2D-4)-(5D^2-6D-8))[x]=\boxed{\begin{aligned}&3(2e^{-2t})+2(-e^{-2t})\\&-(-4e^{-2t})+2(2e^{-2t})\end{aligned}}\ 4$$

$4\quad \underline{(D^2+4D+4)[x]}=x''+4x'+4x=\underline{12e^{-2t}}\ 1\frac{1}{2}$

$2\frac{1}{2}\quad r^2+4r+4=0 \rightarrow r=-2,-2$

$x_h=c_1e^{-2t}+c_2te^{-2t}$

$2\frac{1}{2}\quad x_p=At^2e^{-2t}$

$2\frac{1}{2}\quad x_p'=-2At^2e^{-2t}+2Ate^{-2t}$

$2\frac{1}{2}\quad x_p''=4At^2e^{-2t}-4Ate^{-2t}$
$\qquad\qquad -4Ate^{-2t}+2Ae^{-2t}$

$$\begin{aligned} x_p'' &= 4At^2e^{-2t}-8Ate^{-2t}+2Ae^{-2t}\\ +4x_p' &\quad -8At^2e^{-2t}+8Ate^{-2t}\\ +4x_p &\quad +4At^2e^{-2t}\end{aligned}$$
$2\frac{1}{2}$

$=\underline{2Ae^{-2t}}=12e^{-2t}$ $1\frac{1}{2}$

$2A=12 \rightarrow A=6$

$x=\underline{6t^2e^{-2t}}+\underline{c_1e^{-2t}+c_2te^{-2t}}$
$2\frac{1}{2}\qquad 1\frac{1}{2}$

$-(5D+4)[①]:\ -(5D+4)(2D-2)[x]-(5D+4)(D-2)[y]=-(5D+4)[-e^{-2t}]$

$(2D-2)[②]:\ (2D-2)(5D+4)[x]+(2D-2)(3D+2)[y]=(2D-2)[2e^{-2t}]$

$$(D^2+4D+4)[y]=\boxed{\begin{aligned}&-5(2e^{-2t})-4(-e^{-2t})\\&+2(-4e^{-2t})-2(2e^{-2t})\end{aligned}}\ 4$$

$=\underline{-18e^{-2t}}$ $1\frac{1}{2}$

$y_h=k_1e^{-2t}+k_2te^{-2t}$

$y_p=Bt^2e^{-2t}$

$2B=-18 \rightarrow B=-9$

$y=\underline{-9t^2e^{-2t}}+\underline{k_1e^{-2t}+k_2te^{-2t}}$
$2\frac{1}{2}\qquad 1\frac{1}{2}$

$$x' = \boxed{\begin{aligned} &-12t^2e^{-2t} + 12te^{-2t} \\ &\qquad -2c_2te^{-2t} + c_2e^{-2t} \\ &\qquad\qquad -2c_1e^{-2t} \end{aligned}}$$

6

$$= -12t^2e^{-2t} + (12-2c_2)te^{-2t} + (-2c_1+c_2)e^{-2t}$$

$$2x' = -24t^2e^{-2t} + (24-4c_2)te^{-2t} + (-4c_1+2c_2)e^{-2t}$$

1½

$$-2x \quad -12t^2e^{-2t} \quad -2c_2te^{-2t} \quad -2c_1e^{-2t}$$

1½

$$+y' \quad \boxed{\begin{aligned} &+18t^2e^{-2t} - 18te^{-2t} \\ &\qquad -2k_2te^{-2t} \qquad + k_2e^{-2t} \\ &\qquad\qquad -2k_1e^{-2t} \end{aligned}}$$

4

$$-2y \quad +18t^2e^{-2t} \quad -2k_2te^{-2t} \quad -2k_1e^{-2t}$$

1½

$$= (6-6c_2-4k_2)te^{-2t} + (-6c_1+2c_2-4k_1+k_2)e^{-2t} = -e^{-2t}$$

4

$$6-6c_2-4k_2=0 \qquad -6c_1+2c_2-4k_1+k_2=-1$$

2½

$$k_2 = \frac{6-6c_2}{4} = \frac{3-3c_2}{2}$$

1½

$$\boxed{\begin{aligned} &-6c_1+2c_2-4k_1+\frac{3-3c_2}{2} = -1 \\ &-12c_1+4c_2-8k_1+3-3c_2 = -2 \\ &k_1 = \frac{5-12c_1+c_2}{8} \end{aligned}}$$

2½

$$\boxed{\begin{aligned} x &= 6t^2e^{-2t} + c_1e^{-2t} + c_2te^{-2t} \\ y &= -9t^2e^{-2t} + \frac{5-12c_1+c_2}{8}e^{-2t} + \frac{3-3c_2}{2}te^{-2t} \end{aligned}}$$

2½

[3] $\frac{d^3y}{dx^3} = \frac{d}{dx}\frac{d^2y}{dx^2} = \underline{\frac{d}{dx}\left(v\frac{dv}{dy}\right)} = \underline{\frac{d}{dy}\left(v\frac{dv}{dy}\right)\frac{dy}{dx}}$

$2\frac{1}{2}$ $2\frac{1}{2}$

$= \underline{\left(\frac{dv}{dy}\frac{dv}{dy} + v\frac{d^2v}{dy^2}\right)v}$

4

$\underline{v^2\frac{d^2v}{dy^2} + v\left(\frac{dv}{dy}\right)^2 = yv\frac{dv}{dy}}$

$1\frac{1}{2}$

$\underline{v\frac{d^2v}{dy^2} + \left(\frac{dv}{dy}\right)^2 = y\frac{dv}{dy}}$

$1\frac{1}{2}$

$vv'' + (v')^2 = yv'$

[4] $3r^3-2r^2+12r-8=0 \longrightarrow r^2(3r-2)+4(3r-2)=0 \longrightarrow (r^2+4)(3r-2)=0$

4

1½ $r=\pm 2i,\ \frac{2}{3}$

2½ $x_h = c_1\cos 2t + c_2\sin 2t + c_3 e^{\frac{2}{3}t}$

$$x_{p_1} = [(At+B)\cos 2t + (Ct+E)\sin 2t]t$$

4
$$= (At^2+Bt)\cos 2t + (Ct^2+Et)\sin 2t$$

$$x'_{p_1} = (2At+B)\cos 2t + (-2At^2-2Bt)\sin 2t$$

4
$$+(2Ct^2+2Et)\cos 2t + (2Ct+E)\sin 2t$$

2½
$$= (2Ct^2+(2A+2E)t+B)\cos 2t + (-2At^2+(-2B+2C)t+E)\sin 2t$$

$$x''_{p_1} = (4Ct+(2A+2E))\cos 2t + (-4Ct^2+(-4A-4E)t-2B)\sin 2t$$

6
$$+(-4At^2+(-4B+4C)t+2E)\cos 2t + (-4At+(-2B+2C))\sin 2t$$

2½
$$= (-4At^2+(-4B+8C)t+(2A+4E))\cos 2t + (-4Ct^2+(-8A-4E)t+(-4B+2C))\sin 2t$$

$$x'''_{p_1} = (-8At+(-4B+8C))\cos 2t + (8At^2+(8B-16C)t+(-4A-8E))\sin 2t$$

6
$$+(-8Ct^2+(-16A-8E)t+(-8B+4C))\cos 2t + (-8Ct+(-8A-4E))\sin 2t$$

2½
$$= (-8Ct^2+(-24A-8E)t+(-12B+12C))\cos 2t + (8At^2+(8B-24C)t+(-12A-12E))\sin 2t$$

$$3x'''_{p_1} = (\cancel{-24Ct^2}+(-72A\cancel{-24E})t+(-36B+36C))\cos 2t + (\cancel{24At^2}+(\cancel{24B}-72C)t+(-36A-36E))\sin 2t$$

$$-2x''_{p_1} \quad +(\cancel{8At^2}+(\cancel{8B}-16C)t+(-4A-8E))\cos 2t + (\cancel{8Ct^2}+(16A\cancel{+8E})t+(8B-4C))\sin 2t$$

$$+12x'_{p_1} \quad +(\cancel{24Ct^2}+(24A\cancel{+24E})t+12B)\cos 2t + (\cancel{-24At^2}+(\cancel{-24B}+24C)t+12E)\sin 2t$$

8
$$-8x_{p_1} \quad +(\cancel{-8At^2}\cancel{-8Bt})\cos 2t + (\cancel{-8Ct^2}\cancel{-8Et})\sin 2t$$

6
$$= ((-48A-16C)t+(-4A-24B+36C-8E))\cos 2t + ((16A-48C)t+(-36A+8B-4C-24E))\sin 2t$$

$$= 160t\sin 2t$$

$$\begin{aligned}-48A-16C&=0\\16A-48C&=160\end{aligned} \longrightarrow \begin{aligned}3A+C&=0 \quad ① \longrightarrow C=-3A\\A-3C&=10\\3*① \quad 9A+3C&=0\\10A\qquad&=10 \longrightarrow A=1 \longrightarrow C=-3\end{aligned}$$

$1\frac{1}{2}$ $\quad 2\frac{1}{2}$ $\quad 1\frac{1}{2}$

$$\begin{aligned}-4A-24B+36C-8E&=0\\-36A+8B-4C-24E&=0\end{aligned} \longrightarrow \begin{aligned}-4-24B-108-8E&=0\\-36+8B+12-24E&=0\end{aligned} \longrightarrow \begin{aligned}-24B-8E&=112\\8B-24E&=24\end{aligned}$$

$1\frac{1}{2}$ $\quad 1\frac{1}{2}$

$$\begin{aligned}3B+E&=-14 \quad ②\\B-3E&=3\\3*② \quad 9B+3E&=-42\\10B\qquad&=-39\end{aligned}$$

$2\frac{1}{2}$

$$B=-\frac{39}{10}$$

$$② \quad E=-14-3B=\frac{-140+117}{10}=\frac{-23}{10}$$

$1\frac{1}{2}$

$$x_{p_1}=\left(t^2-\tfrac{39}{10}t\right)\cos 2t+\left(-3t^2-\tfrac{23}{10}t\right)\sin 2t$$

$2\frac{1}{2}$ $x_{p_2}=Gt^2+Ht+J$

$$\begin{aligned}x_{p_2}'&=2Gt+H\\x_{p_2}''&=2G\\x_{p_2}'''&=0\end{aligned}$$

$$\begin{aligned}-8x_{p_2}&=-8Gt^2-8Ht-8J\\+12x_{p_2}'&\qquad\qquad+24Gt+12H\\-2x_{p_2}''&\qquad\qquad\qquad\quad-4G\\3x_{p_2}'''&\qquad\qquad\qquad\quad+0\\&=-8Gt^2+(24G-8H)t\\&\qquad+(-4G+12H-8J)\\&=-80t^2\end{aligned}$$

$2\frac{1}{2}$

$1\frac{1}{2}$

$$\begin{aligned}-8G&=-80 \longrightarrow G=10\\24G-8H&=0 \longrightarrow H=3G=30\\-4G+12H-8J&=0 \longrightarrow J=\tfrac{1}{2}(-G+3H)=40\end{aligned}$$

$$x_{p_2}=10t^2+30t+40$$

$$x=10t^2+30t+40+\left(t^2-\tfrac{39}{10}t\right)\cos 2t+\left(-3t^2-\tfrac{23}{10}t\right)\sin 2t$$
$$+\,c_1\cos 2t+c_2\sin 2t+c_3e^{\frac{2}{3}t}$$

$2\frac{1}{2}$ $\quad 2\frac{1}{2}$ $\quad 1\frac{1}{2}$

[5] $y_1 = x^{-2}, \quad y_2 = x^{-2}e^{\frac{1}{2}x}$

$$W[y_1, y_2] = \begin{vmatrix} x^{-2} & x^{-2}e^{\frac{1}{2}x} \\ -2x^{-3} & -2x^{-3}e^{\frac{1}{2}x} + \frac{1}{2}x^{-2}e^{\frac{1}{2}x} \end{vmatrix} = \frac{1}{2}x^{-4}e^{\frac{1}{2}x}$$

$2\frac{1}{2}$ $\quad 2\frac{1}{2}$

$$g = \frac{x}{2x^2} = \frac{1}{2}x^{-1}$$

$1\frac{1}{2}$

$$y_p = -x^{-2}\int \frac{\frac{1}{2}x^{-1}x^{-2}e^{\frac{1}{2}x}}{\frac{1}{2}x^{-4}e^{\frac{1}{2}x}}\,dx + x^{-2}e^{\frac{1}{2}x}\int \frac{\frac{1}{2}x^{-1}x^{-2}}{\frac{1}{2}x^{-4}e^{\frac{1}{2}x}}\,dx$$

$2\frac{1}{2}$ $\quad 2\frac{1}{2}$

$$= -x^{-2}\int x\,dx + x^{-2}e^{\frac{1}{2}x}\int xe^{-\frac{1}{2}x}\,dx$$

$1\frac{1}{2}$ $\quad 1\frac{1}{2}$

$$\boxed{\begin{array}{ll} x & e^{-\frac{1}{2}x} \\ & + \\ 1 & -2e^{-\frac{1}{2}x} \\ & - \\ 0 & 4e^{-\frac{1}{2}x} \end{array}}$$

$2\frac{1}{2}$

$$= -x^{-2}\left(\frac{1}{2}x^2\right) + x^{-2}e^{\frac{1}{2}x}\left(-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}\right)$$

$1\frac{1}{2}$ $\quad 2\frac{1}{2}$

$$= -\frac{1}{2} - 2x^{-1} \cancel{- 4x^{-2}}$$

$2\frac{1}{2}$

ALREADY IN y_h

$$y = -\frac{1}{2} - 2x^{-1} + c_1x^{-2} + c_2x^{-2}e^{\frac{1}{2}x}$$

$2\frac{1}{2}$